MECHANISM OF MOMENTUM TRANSFER FROM A BODY IN A DEVELOPED FLUIDIZED BED

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The momentum transfer between a fluidized bed and an immersed vertical surface is considered and the force acting on the surface in conditions of intensive particle displacement are estimated.

The interaction of a fluidized bed with immersed bodies is a subject of considerable interest for the hydrodynamic modeling of the bed and also in connection with the analysis of local perturbations of its structure close to various inserted pieces and manufactured components, significantly influencing the external heat and mass transfer. Calculation of the interactive forces is also important when the fluidized bed is used as a heavy medium for gravitational separation and the enrichment of free-flow materials, in the construction of equipment with fluidized beds of various types and specifications, etc.

The introduction of a foreign body in a fluidized bed leads to considerable disruption of the bed structure: On the rear (with respect to the fluidizing flow) part of the body surface there appears a region filled by a slowly slipping dense layer ("cap") of particles [1, 2] and close to the front part of the surface a "free" layer filled by fluidizing medium and containing relatively slight amounts of particles [1, 3]. The presence of this free layer facilitates the generation of bubbles, which pass periodically along the side surface of the body and are completely or partially dislodged from it by the densely packed cap. This structural change affects the flow around the body, in particular changing the mean contact time of various parts of its surface with the dense phase and the bubble phase and the effective force experienced by the surface.

Because there are so many factors that may simultaneously influence the bed interactions with immersed bodies there has been a tendency, in the relatively few investigations of the interactive forces (for example, [2, 4-7]) to isolate the components of the total force due to the flow of the fluidizing medium, the displacement of the disperse phase under the action of the bubbles, the direct impact of particles on the body surface, etc. This analysis, however convenient it may be for the correlation of experimental data, is very arbitrary and does not reflect the real (and whole) picture of the flow of disperse medium around the body, which depends very largely on the size, shape, orientation, and motion of the body and also on the characteristics of the fluidized bed remote from the body.

In constructing a model of the momentum transfer, it is expedient to disregard the features of the process that are specific to particular bodies and instead to concentrate attention on the main mechanism common to different bodies, taking the example of the simplest possible body geometry. The discussion below restricts itself to the momentum transfer with an infinitesimally thin plate, at rest or moving vertically in the bed. In this case, the presence of special regions of altered structure may generally be neglected.

In a bed fluidized by a gas, and also in a fluid bed with developed pulsational motion of the particles, the section of the surface that is in contact with the dense phase of the bed at a particular time evidently plays the main role in the momentum transfer, and the momentum transfer from the sections adjacent to the bubbles moving along the surface may be neglected in the first approximation. This is because the "viscosity" due to the dense phase of the bed is found to be several orders of magnitude higher than the viscosity of the fluidizing agent (see [8], for example) and is analogous to the model in [9], used to describe the external heat transfer. The problem then reduces to the momentum-transfer mechanism from a surface to an adjacent concentrated, macroscopically homogeneous medium. The statistical fractions of the surface area in direct contact with this medium is regarded as an a priori known quantity.

In the analysis of problems of this type, the dense phase is usually modeled by a Newtonian continuous medium, the viscosity of which is estimated by a method analogous to those used in the viscosimetry of homo-

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geneous fluids [8]. For example, the viscosity may be estimated on the basis of experiments with a rotating paddle viscosimeter [10, 11], a descending-sphere viscosimeter [12], a Couette viscosimeter [13-15], and a torsional pendulum [16], and also from an analogy between the behavior of a fluidized bed and a homogeneous viscous liquid. The viscosity has been determined from experiments on the flow of the bed from a nozzle [17], on its motion in a chute [18], and on bubble motion in the bed [19, 20]. There is also a frequency method of viscosity measurement [21]. An example of the use of a continuum model for the calculation of the forces acting on an immersed body may be found in [22].

Such modeling is appropriate for the description of the momentum-flow density in the body of a homogeneous fluidized bed for the dense phase of an inhomogeneous bed close to its boundary. In conditions of developed fluidization the bed may in fact be described as a set of two interacting continua modeling the continuous and disperse phases; the viscosities of the two continua are represented by tensors and they have their origin in momentum transfer by pulsating particles and fluid molecules [23]. For denser systems, in which direct contact between moving particles is significant, the introduction of a single scalar coefficient of viscosity tensor is insufficient: There also appear additional non-Newtonian effects leading, in particular, to the generation of normal "thrust" stresses in the flow [24].*

However, such considerations cannot be used close to the solid boundary of the fluidized bed, obstructing the flow of the fluidizing agent and changing the structure and properties of the disperse system in the layer at the wall (the thickness of which should be of the order of a few particle diameters) in comparison with the structure and properties far from the boundary. It is clear that, in general, the hydrodynamic equations and rheological relations remote from the immersed body do not hold inside this layer, which may play a very large role in the mass transfer to interchanging elements of the disperse phase at the body surface. In fact, this process is essentially unsteady: The exchange of these elements or of individual particles in the composition of a single element occurs very rapidly in conditions of developed fluidization, so that in the characteristic time of this exchange the perturbation from the body is practically unable to pass beyond the limits of the boundary layer. Therefore, it is impossible to regard as adequate an approach, for example, in which the nonsteady momentum transfer to an element of the dense phase (a "packet" of particles) making contact with the surface is considered in a continuum model, when this element is described as a homogeneous continuous medium. Correspondingly, the viscosity values obtained in various experiments based on relations for homogeneous Newtonian fluids should be different, as is in fact observed (see [8, 10-21] and also the comparison in [25]). Here once again there is a clear analogy with the situation discussed in [9].

As in [9], it is now assumed that the particles are involved in developed pulsational (pseudoturbulent) motion; i.e., there is ideal mixing of the particles and, in particular, the mean time τ spent by any particle in the direct vicinity of the plate is small. Theoretical estimates based on the results of [23], and also some experimental data indicate that the characteristic frequency $f=\tau^{-1}$ of small-scale perturbations of individual particles is very large and considerably exceeds the frequency of bubble generation at the plate. Therefore, nonsteady effects associated with the exchange of dense-phase volumes may be completely disregarded. The difference in the characteristic times of the small-scale pulsations and the observed macroscopic motion of bubbles and large groups of particles was discussed sufficiently thoroughly in [26].

The impermeability of the plate for the particles is due, as is known, to the existence of a boundary layer of increased porosity and hence the possible escape of excess volume of fluidizing agent along the plate. This escape (observed in [27, 28], for example) may, in principle, have a considerable effect on the conditions of momentum transfer and so complicates the analysis. However, following [9], this effect will, for simplicity, be neglected in the present analysis. Then, if the particles did not pulsate and there were no friction of the fluidizing agent and the particles with the plate, the plate would in general have no effect on the bed. In reality, the medium is entrained by the plate and its mean velocity within some thin surface layer diverges from the fluidization velocity. The "excess" momentum of the medium is transferred by particles within this layer;

*While considering attempts to determine the viscosity of a fluidized bed and of other types of disperse system, it is appropriate to note a very common error. It is assumed that when there is no contact between grains or when there is a fluid "lubricant" separating adjacent grains, the internal friction of the disperse phase is identically zero or very small. In fact, of course, this is not true. The internal friction characterizes the density of the whole disperse-phase momentum flow through some imagined area within the bed, and this flow is due not only (or mainly, as a rule) to the momentum transfer in direct contacts between particles but to momentum transfer by the pulsational motion of particles, as is the case in gases whose molecules perform thermal motion, in turbulent media with pulsating liquid molecules, and in other classical systems.

if the pulsational motion of the particles is sufficiently intense, particle transfer between this layer (the thickness of which may be estimated on the basis of the analysis below) and the body of the bed may be regarded as instantaneous. Thus, according to the model developed here, the momentum transfer from the plate and the entrained volumes of fluidizing agent is mainly determined by the acceleration of the fluidizing-agent particles arriving at the surface layer from the body of the bed, which constitute an effective momentum sink. Momentum transfer as a result of direct collision between the particles and the plate surface is neglected here, on the assumption that the change in particle velocity is due entirely to interactions with the fluidizing medium. The above considerations are analogous to those used in [9], and the definite successes achieved in [9] in describing heat transfer may be regarded as indirect confirmation of their truth.

Note that rapid particle exchange in the surface layer and the accompanying momentum transfer to the body of the bed prevent the formation of the usual two-phase boundary layer at a plate, facilitating its constant disruption.

The unperturbed state remote from the plate ($y \to \infty$) is described by the equations

$$-\frac{dp_\infty}{dx} - d_0 g - F(\rho, v_\infty) = 0, \quad -\rho(d_1 - d_0)g + F(\rho, v_\infty) = 0, \tag{1}$$

for which p_∞ and v_∞ may be determined as a function of ρ and a parameter. In a homogeneous medium ρ characterizes the true mean bulk concentration of disperse phase in the bed, while in an inhomogeneous medium it characterizes only the dense phase.

Close to the plate the velocity of the fluidizing medium is written in the form $v + v_\infty$. Then on the basis of the assumptions and the model outlines above, the following relations may be written for v:

$$\mu \frac{d^2 v}{dy^2} - P = 0, \quad v = V - v_\infty (y = 0), \quad v \to 0 \, (y \to \infty), \quad \mu = \mu_0 G(\rho), \tag{2}$$

where the momentum-sink density may be written, as in [9], in the form

$$P = fnmW(\tau), \quad f = \frac{1}{\tau}, \quad n = \rho\left(\frac{4}{3}\pi a^3\right)^{-1}, \quad m = \frac{4}{3}\pi a^3 d_1. \tag{3}$$

The function $G(\rho)$ characterizes the difference between the effective viscosity of a fluid filtering in a mobile porous body formed by particles and the physical viscosity of the fluid [29, 30]. In the general case, by a simple redefinition, $G(\rho)$ may also take into account the momentum transfer in the fluidizing medium due to random pulsational motion, considered in [23]. In Eqs. (2) and (3) it is assumed that the particles arrive at an arbitrary point of the surface layer with the same probability and have zero velocity; after a time τ they are transferred instantaneously to the body of the bed, acquiring a velocity $W(\tau)$, which depends on the position of the point, of course. In principle, Eq. (3) is easily generalized to the situation in which the time τ is a random quantity with a homogeneous distribution function.

The variable $W(t)$ is determined using the equation of motion of a particle, taking into account the restriction of the flow around it. For unit mass of disperse phase, this equation may be written in the form

$$\frac{dW}{dt} = \alpha\left[(v_\infty + v - W)^2 \operatorname{sign}(v_\infty + v - W) - v_\infty^2\right] + \beta(v - W) \tag{4}$$

(the effective particle weight is expressed here in terms of the hydraulic force corresponding to the velocity v_∞). The coefficients α and β will be represented in a form derived from the semiempirical formula of [31]:

$$\alpha = \frac{3.5}{4\varepsilon a}\frac{d_0}{d_1}, \quad \beta = \frac{75}{2}\frac{\rho}{\varepsilon^2}\frac{\mu_0}{a^2 d_1}, \quad \varepsilon = 1 - \rho \tag{5}$$

approximately correct when $\rho \geqslant 0.03$. The force F in Eq. (1) then takes the form

$$F(\rho, v) = nm(\alpha v^2 + \beta v) = \rho d_1(\alpha v^2 + \beta v). \tag{6}$$

Solving Eq. (4) with the initial condition $W(0) = 0$, it is easy to find $W(t)$ and P from Eq. (3) and then, solving Eq. (2), to determine the velocity of the fluidizing agent at the plate and calculate the tangential stress acting on it. The disperse-phase velocity may be calculated as follows:

$$w(y) = \frac{1}{\tau}\int_0^\tau W(t)\,dt. \tag{7}$$

The dependence on y in this formula is through $v(y)$, which appears in the expression for $W(t)$. The order of magnitude of τ is estimated as follows: [9, 23]

$$\tau \approx \frac{\delta}{u'} = \frac{C\delta}{v_\infty}, \quad C = C(\rho), \tag{8}$$

where δ is the thickness (determined below) of the surface layer in which $v(y)$ and $w(y)$ are significantly nonzero, and the coefficient $C(\rho)$ was investigated for small particles in [23]. For large particles, C must in general be regarded as an empirical parameter.

As the necessary computations are very complicated, consideration is restricted below to the special cases corresponding to small and large particles.

Small Particles. The hydraulic drag of a bed of sufficiently small particles is a linear function of the velocity, i.e., in Eq. (4) the term proportional to α may be neglected. Then Eqs. (1)-(4) give successively

$$W(t) = v(1 - e^{-\beta t}), \quad P = (\rho d_1/\tau)(1 - e^{-\beta\tau})\, v; \tag{9}$$

$$v(y) = (V - v_\infty)\, e^{-\lambda y}, \quad \lambda^2 = \frac{\rho d_1}{\tau \mu_0 G}(1 - e^{-\beta\tau}), \tag{10}$$

and for the tangential stress at the wall

$$T = \mu dv/dy|_{y=0} = -\mu_0 G\lambda (V - v_\infty). \tag{11}$$

Using Eq. (5) and the expression

$$v_\infty = \frac{(d_1 - d_0)\, g}{d_1 \beta} \tag{12}$$

derived from Eq. (1), and setting $\delta \approx \lambda^{-1}$, an equation for τ is obtained

$$\tau = \frac{z}{\beta}, \quad [z(1 - e^{-z})]^{1/2} \approx \left(\frac{75}{2}\right)^{3/2} \frac{\rho C G^{1/2}}{\varepsilon^3} \, \frac{\mu_0^2}{a^3 d_1 (d_1 - d_0)\, g}. \tag{13}$$

The unique solution of this equation finally determines the field $v(y)$ and the tangential stress T. These expressions may readily be written in simple analytic form in the limiting cases of small and large z. For $z \ll 1$, $1 - e^{-z} \approx z$ and, in general, λ and T in Eqs. (10) and (11) are independent of τ:

$$\lambda \approx \left(\frac{75}{2G}\right)^{1/2} \frac{\rho}{\varepsilon a}, \quad T \approx -\left(\frac{75G}{2}\right)^{1/2} \frac{\rho\mu_0}{\varepsilon a}(V - v_\infty). \tag{14}$$

For $z \gg 1$, $e^{-z} \approx 0$ and from Eqs. (10)-(13)

$$\lambda \approx \frac{2\varepsilon^2}{75CG} \frac{a^2 d_1 (d_1 - d_0)\, g}{\mu_0^2}, \quad T \approx -\frac{2\varepsilon^2}{75C} \frac{a^2 d_1 (d_1 - d_0)\, g}{\mu_0}(V - v_\infty). \tag{15}$$

The total force acting on the plate, the area of both sides of which is S, may evidently be written in the form

$$\Phi = sST = -kS(V - v_\infty), \tag{16}$$

where, from Eqs. (14) and (15), the drag coefficient k for small and large z may be written as follows:

$$k \approx \left(\frac{75G}{2}\right)^{1/2} \frac{s\rho\mu_0}{\varepsilon a} (z \ll 1), \quad k \approx \frac{2\varepsilon^2 s}{75C} \frac{a^2 d_1 (d_1 - d_0)\, g}{\mu_0} (z \gg 1). \tag{17}$$

It is evident from Eq. (16) that the force Φ is a linear function both of the plate area and of its velocity, however large this velocity may be. This important result was first obtained experimentally in [32].

It is simple to show, estimating the right-hand side of Eq. (13), that the parameter z may in fact be small for beds fluidized by gas and large in comparison with unity for beds fluidized by liquid drops.

The dependence of k on physical parameters is found to be very complex and unusual. For $z \ll 1$ (typical of fluidization by a gas) the drag coefficient rises linearly with increase in gas viscosity, is inversely proportional to the particle size, and is independent of the phase density. For $z \gg 1$, on the other hand, it decreases in inverse proportion to rise in velocity, is proportional to the square of the particle size, and depends on the density of the particles and the fluidizing agent. For $z \sim 1$, dependences of intermediate types would be expected, valid in a limited parameter range; for example, there is a region in which k is practically independent of viscosity.

The dependence of k on the porosity is also found to be complex. Using the estimate of C in [23], it is found that, in the porosity range of main interest, k decreases with increase in porosity when $z \ll 1$ and increases when $z \gg 1$.

Large Particles. In this case the term with the factor β may be neglected in Eq. (4). For upward motion of the plate with velocity $V > 0$, it is always the case that $v_\infty + v - W > 0$. Calculations lead to the following solution of Eq. (4) with zero initial conditions:

$$W(t) = \frac{2(e^{\sigma t}-1)v_\infty + (e^{\sigma t}-1)v}{2e^{\sigma t}v_\infty + (e^{\sigma t}-1)v}\, v, \quad \sigma = 2av_\infty. \tag{18}$$

Random pulsations of large particles are usually very intensive, i.e., τ may be assumed small. For simplicity, only the important limiting case when $\sigma\tau \ll 1$ will be considered here. Then the relation in Eq. (2) takes the form

$$\frac{d^2v}{dy^2} - \frac{\rho d_1 \sigma}{2\mu_0 G}\,\frac{2v_\infty + v}{v_\infty}\, v = 0 \tag{19}$$

and has the first integral

$$\frac{dv}{dy} = -v(p+qv)^{1/2}, \quad p = \frac{\rho d_1 \sigma}{\mu_0 G}, \quad q = \frac{\rho d_1 \sigma}{3\mu_0 G v_\infty} \tag{20}$$

(to determine the arbitrary constants it is assumed that v and dv/dy vanish simultaneously as $y \to \infty$). The velocity field $v(y)$ is determined from a relation obtained by integrating Eq. (20) with the boundary condition for $y = 0$ from Eq. (2)

$$\frac{(p+qv)^{1/2} - p^{1/2}}{(p+qv)^{1/2} + p^{1/2}} = \frac{[p + q(V - v_\infty)]^{1/2} - p^{1/2}}{[p + q(V - v_\infty)]^{1/2} + p^{1/2}} \exp(-p^{1/2} y). \tag{21}$$

The tangential stress may be expressed using **Eq. (20), also taking into account** the boundary condition in Eq. (2). Simple calculation using the formula given earlier for the force Φ acting on the plate leads to Eq. **(16)** with drag coefficient

$$k = (3.5)^{1/4} s(\rho G)^{1/2}\left[\frac{d_0(d_1 - d_0)\mu_0^2 g}{\varepsilon a}\right]^{1/4}\left(1 + \frac{V - v_\infty}{3v_\infty}\right)^{1/2}. \tag{22}$$

In this case, evidently, the drag coefficient depends (though not very strongly) on the ratio between the phase velocity and the velocity of the fluidizing agent in the imperturbed dense phase. The dependence of k on the porosity and the physical parameters following from Eq. (22) is also very weak. Equation (22) is universal in the sense that it does not include any parameters characterizing the pulsational motion of the particles.

In the case of downward motion of the plate ($V < 0$), there is always a region $0 < y < y_*$ at its surface in which $v_\infty + v < 0$. Within this region $v_\infty + v - W < 0$ at times satisfying the condition $0 < t < t_*(y)$. For these times, when $\beta = 0$, Eq. (4) gives the relation

$$\operatorname{arctg}\frac{v_\infty + v - W}{v_\infty} = -\frac{1}{2}\sigma t + \operatorname{arctg}\frac{v_\infty + v}{v_\infty}, \quad t < t_*(y). \tag{23}$$

The value of $t_*(y)$ is found from the condition $W(t_*) = v_\infty + v$, i.e., Eq. (23) gives

$$t_*(y) = -\frac{2}{\sigma}\operatorname{arctg}\frac{v_\infty + v(y)}{v_\infty}. \tag{24}$$

When $t > t_*(y)$, $v_\infty + v - W > 0$ and solving Eq. (4) with the initial condition $W(t_*) = v_\infty + v$ leads to the result

$$W(t) = v + \frac{2}{e^{\sigma t} + 1} v_\infty, \quad t > t_*(y). \tag{25}$$

Let V be small in absooute magnitude, so that $\tau > t_*(0) = \max t_*$; as before, $\sigma\tau \ll 1$. This leads to the result

$$\frac{|V|}{v_\infty} < \operatorname{tg}\frac{\sigma\tau}{2} \approx \frac{\sigma\tau}{2} \ll 1, \tag{26}$$

and when this condition is satisfied, Eq. (19) is replaced, in the region $0 < y < y_*$, by the relation

$$\frac{d^2v}{dy^2} - \frac{\rho d_1}{\tau\mu_0 G}(v_\infty + v) = 0 \tag{27}$$

and likewise Eq. (20) is replaced by the relation

$$\frac{dv}{dy} = \left[\frac{2\rho d_1}{\tau\mu_0 G}\, v\left(v_\infty + \frac{1}{2}v\right) + D\right]^{1/2}, \tag{28}$$

where D is an arbitrary constant. As before, Eq. (20) holds in the region $y > y_*$. If dv/dy is required to be continuous at $y = y_*$ and the definition $v_\infty + v(y_*) = 0$ is introduced, calculations taking into account Eq. (26) give

$$D = \frac{\rho d_1 v_\infty^2}{\mu_0 G}\left(\frac{1}{\tau} + \frac{2}{3}\sigma\right) \approx \frac{\rho d_1 v_\infty^2}{\tau \mu_0 G}. \tag{29}$$

Using this expression in Eq. (28) and taking dv/dy at y = 0, where $v(0) = V - v_\infty$, leads again to a formula for Φ:

$$\Phi = -kSV, \quad k = s\left(\frac{\rho d_1 \mu_0 G}{\tau}\right)^{1/2}. \tag{30}$$

It is evident that when $\sigma\tau \ll 1$ the drag is considerably larger for downward motion of the plate than for upward motion. For V = 0 Eq. (30) gives, formally, Φ = 0, which contradicts the conclusion obtained from Eqs. (16) and (22). This contradiction is only apparent: It is due to the use of the approximate equality in Eq. (29). The true value of the force acting on a plate at rest is obtained from Eqs. (16) and (22) at V = 0 and is nonzero.

Note that, in contrast to the drag coefficient for upward motion in Eq. (22), the drag coefficient in Eq. (30) describing downward motion depends significantly on the time τ, and hence on the characteristics of the pulsational motion. Expressing τ in accordance with Eq. (8) it is simple to write k as a function of the physical parameters.

When the first inequality in Eq. (26) is violated, there is a subregion $0 < y < y_*'$ of the region $0 < y < y_*$, in which Eq. (23) must be used instead of Eq. (25) to calculate W(τ); y_*' is determined from the obvious condition $t_*(y_*') = \tau$. It is also simple to generalize the results to the situation when $\sigma\tau \gtrsim 1$, but the computations become more complicated.

Note that in an inhomogeneous bed s may be determined from an approximate relation deriving from two-phase fluidization theory, as proposed in [9]. Comparison of the results of [9] with those of the above theory indicates that there is a definite analogy between the processes of momentum and heat transfer for the immersed body.

If the formation of singular regions at the surface of the body immersed in the fluidized bed is neglected, a similar method may be used to calculate the forces acting not only on a vertical plate but also on a body of different shape. The values of s, ρ, and v_∞ must then be assumed to depend on the position of the element of body surface considered. For example, in the simplest model of an incompressible homogeneous bed (ρ = const, s = 1) the value of v_∞ may be obtained by solving the "external" problems of the potential flow around the body of a continuous medium modeling the disperse phase and of the filtration of the fluidizing agent in the mobile porous body formed by this phase, as the limit of the tangential fluidizing-agent velocity on approaching the body surface. This value is then used as the external limit for an internal solution v(y) of the type considered above, valid in the thin surface layer; the relation with the basic ideas of the method of asymptotic-series expansions is here completely evident.

Direct comparison of the results obtained with experimental data (taken, in particular, from the works cited above) is difficult for two reasons. Firstly, these data usually refer to the "viscosity" of the fluidized bed determined from an inadequate analogy with the viscosimetry of homogeneous Newtonian media. As follows from the analysis given above, the momentum-transfer mechanism in a fluidized bed is fundamentally different from that operating in a homogeneous fluid (for example, no boundary layer of the usual type is formed at a plate in a fluidized bed), and the introduction of such a "viscosity" as a universal characteristic of the bed is physically meaningless. Secondly, as follows from the theory, the dependence of the drag forces on the fluidizing-agent viscosity, particle size, etc., in various ranges of parameters, may differ greatly; this explains, to some extent, the contradictions and disagreements in the experimental results obtained by different experimenters. Verification of the results evidently requires the formulation of special and more accurate experiments.

In conclusion, the two main limitations of the above theory should be emphasized. Firstly, direct momentum transfer between the body surface and adjacent particles and momentum transfer between contacting particles have been neglected. Of course, these assumptions cannot be correct for fluidization numbers close to unity, when there is a momentum-transfer mechanism of the type discussed in [24]. However, in conditions of developed fluidization, these assumptions are approximately correct, as is evident, for example, in that the "viscosity" determined for beds of smooth and rough or angular particles is found to be almost the same [8]. Secondly, the possible escape of fluidizing agent through a layer of increased porosity at the plate surface [27] has been neglected. Semiempirical allowance for this effect is possible if modified (for example, on the basis of experiment) relations for the porosity and fluidizing-agent velocity in the direct vicinity of the surface are used.

NOTATION

a, particle radius; C, function appearing in Eq. (8); D, arbitrary constant in Eq. (28); d_0, d_1, fluidizing-agent and particle-material density; F, interactive force between continuous and disperse phases; f, pulsation frequency; G, function appearing in Eq. (2); g, acceleration due to gravity; k, plate drag coefficient; m, plate mass; n, number of particles per unit volume; P, specific power of momentum sinks in surface region; p, q, coefficients appearing in Eq. (20); p_∞, unperturbed pressure; S, plate area; s, statistical fraction of contact time with dense phase; T, tangential stress; t, time; t_*, critical particle-takeoff time; V, plate velocity; v, discrepancy of fluidizing-agent velocity in the interstices between particles from its unperturbed value v_∞; W, particle velocity; w, mean disperse-phase velocity; x, y, longitudinal and transverse coordinates; y_*, y_*', critical values of y; α, β, coefficients appearing in Eq. (5); δ, surface-layer thickness; ε, porosity; λ, exponential factor in Eq. (10); $\mu=\mu_0 G$; μ_0, fluidizing-agent viscosity; ρ, bulk concentration of particles in dense phase; σ, exponential factor in Eq. (18); τ, residence time of particle in surface layer; Φ, force acting on plate.

LITERATURE CITED

1. D. H. Glass and D. Harrison, Chem. Eng. Sci., 19, 1001 (1964).
2. A. P. Baskakov, B. V. Berg, and V. V. Khoroshavtsev, Teor. Osn. Khim. Tehnol., 5, 885 (1971).
3. V. N. Korolev and N. I. Syromyatnikov, Dokl. Akad. Nauk SSSR, 203, 58 (1972).
4. H. Reuter, Chem.-Ing. Tech., 38, 880 (1966).
5. R. B. Rozenbaum, O. M. Todes, and L. N. Fainshtein, Inzh.-Fiz. Zh., 25, 601 (1973).
6. B. A. Michkovskii and A. P. Baskakov, in: Heat and Mass Transfer and Nonequilibrium Thermodynamics of Disperse Systems [in Russian], UPI, Sverdlovsk (1974), p. 163.
7. B. A. Michkovskii and V. A. Kirakosyan, in: Industrial Boiling-Bed Furnaces [in Russian], UPI, Sverdlovsk (1976), p. 53.
8. K. Schügerl, in: Fluidization [in Russian], Khimiya, Moscow (1974), p. 228.
9. Yu. A. Buevich, D. A. Kazenin, and N. N. Prokhorenko, Inzh.-Fiz. Zh., 29, 410 (1975).
10. I. Furukawa and T. Ohmae, Ind. Eng. Chem., 50, 821 (1958).
11. W. W. Schuster and F. C. Haas, J. Chem. Eng. Data, 5, 525 (1960).
12. H. Trawinski, Chem.-Ing. Tech., 25, 201, 229 (1953).
13. F. Fa-key Liu and C. Orr, J. Chem. Eng. Data, 5, 430 (1960).
14. K. Schügerl, M. Merz, and F. Fetting, Chem. Eng. Sci., 15, 1 (1961).
15. H. Ritzman and K. Schügerl, Chem. Eng. Sci., 29, 427 (1974).
16. T. Hagyard and A. M. Sacerdote, Ind. Eng. Chem. Fund., 5, 500 (1966).
17. P. Rebu, Boiling Beds [in Russian], TsIINTsvetmash, Moscow (1956).
18. J. S. M. Botteril and M. Kolk, Chem. Eng. Progr. Symp. Ser., 67, 70 (1971).
19. J. D. Murray, Rheol. Acta, 6, 27 (1967).
20. J. R. Grace, Can. J. Chem. Eng., 48, 30 (1970).
21. G. I. Lapshenkov, N. I. Gel'perin, and V. G. Ainshtein, in: Chemical-Engineering Processes and Equipment [in Russian], Khimiya, Moscow (1967), p. 47.
22. Yu. A. Buevich, Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza, No. 4, 94 (1966).
23. Yu. A. Buevich, J. Fluid Mech., 56, 313 (1972).
24. M. A. Gol'dshtik and B. N. Kozlov, Zh. Prikl. Mekh. Tekh. Fiz., No. 4, 67 (1973).
25. W. Siemes and L. Hellmer, Chem. Eng. Sci., 17, 555 (1962).
26. O. M. Todes, A. K. Bondareva, and M. B. Grinbaum, Khim. Prom-st, No. 6, 8 (1966).
27. V. N. Korolev and N. I. Syromyatnikov, Zh. Vses. Khim.-O-va, 15, 585 (1970).
28. N. B. Kondukov, L. I. Frenkel', S. A. Nagornov, N. Ya. Romanenko, and V. P. Tarov, Dokl. Akad. Nauk SSSR, 224, 1138 (1975).
29. T. S. Lundgren, J. Fluid Mech., 51, 273 (1972).
30. Yu. A. Buevich, Z. Angew. Math. Mech., 56, 379 (1976).
31. S. Ergun, Chem. Eng. Progr., 48, 89 (1952).
32. R. B. Rozenbaum and O. M. Todes, in: Heat and Mass Transfer [in Russian], Vol. 6, Minsk (1976), p. 98.